Comments on : Frame Dragging Anomalies for Rotating Bodies

Hristu Culetu,
Ovidius University, Dept.of Physics,
B-dul Mamaia 124, 8700 Constanta, Romania,
e-mail : hculetu@yahoo.com

December 28, 2008

Abstract

It is shown that Collas and Klein (ArXiv : 0811.2471 [gr-qc]) wrongly concluded that "negative frame dragging" phenomenon takes place at all finite r and z coordinate values . We argue that a test particle with zero angular momentum counter-rotates with respect to the source in the "time machine" region only. In addition, Bonnor's spacetime has an event horizon at $r_H = 0$.

Keywords : event horizon, time machine region, frame dragging.

These Comments concern the anomalous "negative frame dragging" phenomenon which appears, according to Collas and Klein [1], whenever "zero angular momentum test particles acquire angular velocity in the opposite direction of rotation from the source of the metric".

We argue that the "Proposition 1" of pp. 4 is partially incorrect. Collas and Klein state that "... $\omega \prec 0$ everywhere in S_{B0}", probably on the grounds that $L \succ 0$ in their Eq. (7) leads to $\omega \prec 0$. But that is valid only when M (or n) $\succ 0$, where n is given in Eq. (9). But why n must be positive ? In authors' opinion, h (with dimension of length squared) is a parameter related to rotation. We know there are two directions of rotation and therefore h may be negative, too.

In the revised version [2], Collas and Klein justify their choice, $h \succ 0$, stating that "we assume, without loss of generality, as in [3], that $h \succ 0$ ". But exactly the sign of h (or of M) leads to the so called "negative frame dragging" effect. Therefore, we consider it is not a physical phenomenon produced by the choice of the sign of h.

Let us notice that, when we pass from the Minkowski spacetime

$$ds^2 = -dT^2 + dR^2 + dZ^2 + R^2 d\Phi^2 \tag{0.1}$$

to the uniformly rotating one [4]

$$ds^2 = -(1 - \Omega^2 R'^2)dT'^2 + dR'^2 + dZ'^2 + 2\Omega R'^2 d\Phi' dT' + R'^2 d\Phi'^2 \tag{0.2}$$

by means of the coordinate transformation

$$\Phi' = \Phi - \Omega T, \quad T' = T, \quad Z' = Z, \quad R' = R, \tag{0.3}$$

the sign of the metric coefficient $g_{\Phi' T'}$ changes when the direction of rotation is reversed.
A similar effect takes place on M in Eq. (1) of Ref. [1] : it could have both signs. Therefore, in our opinion, we have $\omega \prec 0$ only when $r \prec n$ (the "time machine " region), where closed timelike curves (CTC) are possible.
In fact, even the authors of [1] recognize at pp. 6, at the end of Chap. 3, that "the sign of the metric coefficient L determines the sign of the frame dragging ω". In other words, $L \prec 0$ (or $r \prec \sqrt{2|h|}$) leads to $\omega \prec 0$ and not M. Similar conclusions were reached in [5]. If we divide the two relations from Eq. (5.5) of Ref. [5] (with $\omega = 0$), one obtains

$$\frac{\dot{\phi}}{\dot{t}} = \frac{d\phi}{dt} = \frac{L + bE}{(r^2 - b^2)E - bL} \tag{0.4}$$

i.e. exactly Eq. (6) of Ref. [1], with $F = 1$, L instead of p_ϕ and $(-b)$ instead of M . Taking above a zero angular momentum particles , one get

$$\frac{d\phi}{dt} = \frac{b}{r^2 - b^2} \tag{0.5}$$

Here b is considered to be positive since its sign depends upon how we define the "improper" time translation in Eq. (2.2), Ref. [5]. In conclusion, in our view, the negative value of ω in (7), Ref. [1] has nothing to do with region S_{B0} but comes from the negative value of $g_{\phi\phi}$ (the time machine region $r \prec b$ in [5]). Its boundary $r = b$ is the velocity of light surface. Because $g_{\phi t} \neq 0$ when $g_{\phi\phi} = 0$, the metric (2.5) (with $\omega = 0$) from [5] is nonsingular at $r = b$. Therefore, the timelike curves may cross into the time machine region and viceversa [6].

One should finally mention the problem of the existence of an event horizon in Bonnor's dust metric. Collas and Klein argued in Chap.4 ("Concluding remarks" of [7]) that :
"The spacetime considered here has some unrealistic features. It has an isolated singularity with no event horizon".
But their spacetime (1) of [7] (and even the metric (1) of Ref. [1]) has an event horizon which is obtained from

$$g_{tt} - \frac{g_{t\phi}^2}{g_{\phi\phi}} = 0, \tag{0.6}$$

(see Eq. (7) of [8] or Eq. (7) of [9]). Eq. (6) leads to $r_H = 0$, i.e. the horizon is located on the rotation axis, in the interior of the time machine region. The fact that the numerator from the l.h.s. of (6) equals r^2 represents the Collas and Klein "coordinate condition" (in [1]) or "gauge condition" in [7]).

References

[1] P. Collas and D. Klein, ArXiv : 0811.2471 v1 [gr - qc].

[2] P. Collas and D. Klein, ArXiv : 0811.2471 v2 [gr - qc].

[3] B. R. Steadmann, Gen. Rel. Grav. 35, 1721 (2003).

[4] D. Soler, ArXiv : gr-qc/0511041.

[5] H. Culetu, AIP Proceedings of the 2nd International Conference on the Dark Side of the Universe, vol. 878, pp. 330, Madrid, Spain, 2006 ; ArXiv : hep-th/0602014 v2.

[6] M. Cvetic et al., ArXiv : hep-th/0504080.

[7] P. Collas and D. Klein, ArXiv : 0811.2468 [gr - qc].

[8] H. Culetu, 12th Conference on Recent Developments in Gravity, NEB XII, Jour. of Phys. : Conference Series 68 (2007) 012036.

[9] H. Culetu, XXIX th Spanish Relativity Meeting (ERE 2006), Jour. of Phys. : Conference Series 66 (2007) 012055.